**FPT University**

**Course: Discrete mathematics**

**Course ID: MAD101**

**Student’s name: Group:**

**EXERCISES - CHAPTER 1**

**PART I (7 MARKS)**

1. Determine whether these statements are true or false.
2. **If** 1 + 1 = 3, **then** unicorns exist.
3. 1 + 1 = 2 **if and only if** 2 + 3 = 4.
4. **If** 1 + 1 = 3, **then** dogs can ﬂy.
5. 1 + 1 = **3 if and only if** monkeys can ﬂy.
6. 0 > 1 **if and only if** 2 > 1.
7. How many rows appear in a **truth table** for each of these **compound propositions**?
8. (q → ¬p) ∨ (¬p → ¬q)
9. (p ∨ ¬t) ∧ (p ∨ ¬s)
10. (p → r) ∨ (¬s → ¬t) ∨ (¬u → v)
11. (p ∧ r ∧ s) ∨ (q ∧ t) ∨ (r ∧ ¬t)
12. Construct a **truth table** for each of these compound propositions.
13. p → p ∧ q
14. p → p ∨ q
15. p → r
16. ¬r → ¬p
17. ¬p → ¬r
18. (a) Find a proposition with the given **truth table**.

|  |  |  |
| --- | --- | --- |
| *p* | *q* | ? |
| T | T | F |
| T | F | F |
| F | T | T |
| F | F | F |

(b) Find a proposition using only *p**q*, and the connective  that has this **truth table**.

1. Write a proposition **equivalent** to *p*  *q* that uses only *p**q* and the connective .
2. Write a proposition **equivalent** to *p*  *q* using only *p**q* and the connective .
3. Show that each of these conditional statements is a **tautology** by using truth tables.
4. p∧q → p (Simpliﬁcation)
5. p → p ∨ q (Addition)
6. [¬p ∧ (p ∨ q)] → q (Disjunctive syllogism)
7. (p → r) ∧¬r] → ¬p (Modus tollens)
8. [p ∧ (p → q)]→ q (Modus Ponens)
9. Use truth tables to verify these **equivalences**.
10. p ∧ T ≡ p
11. p ∨ T ≡ T
12. p ∨ p ≡ p
13. ¬(¬p) ≡ p
14. ¬(p ∧ q) ≡¬p ∨¬q (De Morgan’s Law)
15. Use **De Morgan’s laws** to ﬁnd the negation of each of the following statements.
16. Minh knows Java and C++.
17. Rita will move to Oregon or Washington.
18. Write the **negation** of the following statements. (Don't write “It is not true that *…*.”)
19. It is Thursday and it is cold.
20. If it is rainy, then we go to the movies.
21. Khai or Anh are absent.
22. If it is sunny, then it is hot.
23. Let W(x) be the statement “the word x contains the letter a.” What are these truth values? W(orange)
    1. W(lemon)
    2. W(true)
    3. W(false)
24. Determine the **truth value** of each of these statements if the domain for all variables consists of real numbers.
25. ∀x(x > 1 → x2 > 1)
26. ∀x(x > 1 ∧ x2 > 1)
27. ∀x(x > 1 ∨ x2 > 1)
28. Express the **negations** of each of these statements.
29. ∀x(P(x) ∧ Q(x))
30. ∀x(P(x) ∨ Q(x))
31. ∀x(P(x) → Q(x))
32. ∀x∃y(P(x,y) → Q(x, y))
33. Suppose the variable *x* represents students and *y* represents courses, and *F*(*x*): *x* is a freshman *A*(*x*): *x* is a part-time student *T*(*x**y*): *x* is taking *y*.

Write the statement using these predicates and any needed quantifiers.

1. Mikko is a freshman
2. Joe is not taking any course.
3. Some part-time students are not freshmen.
4. Every freshman is taking at least one course.
5. For each of these **arguments** determine whether the argument is correct or incorrect and explain why.
6. Everyone enrolled in the FPT university has lived in a dorm. Van has never lived in a dorm. Therefore, Van is not enrolled in the university.
7. A sport car is fun to drive. Tri’s car is not a sport. Therefore, Tri’s car is not fun to drive.
8. Every IA major takes discrete mathematics. Cuong is taking discrete mathematics. Therefore, Cuong is an IA major.
9. All parrots like fruit. My pet bird is not a parrot. Therefore, my pet bird does not like fruit.
10. Everyone who eats granola every day is healthy. Linda is not healthy. Therefore, Linda does not eat granola every day.

**PART II (3 MARKS)**

1. Write each of these statements in the form **“if p, then q”** in English. [Hint: Refer to the list of common ways to express conditional statements provided in this section.]
2. It is **necessary** to wash the boss’s car to get promoted.
3. A **sufﬁcient condition** for the warranty to be good is that you bought the computer less than a year ago.
4. Willy gets caught **whenever** he cheats.
5. You can access the website **only if** you pay a subscription fee.
6. Carol gets seasick **unless** she is not on a boat.
7. Determine whether these pairs of **compound propositions** are **logically equivalent** or not.
8. p ∧ (p ∨ q) and p
9. (p ∧ q) v r and p ∧ (q v r)
10. p → r and p → ¬r
11. p → q and ¬q → ¬p
12. Suppose that the domain of the propositional function P(x) consists of the integers 1, 2, 3, and 4. Express these statements without using quantiﬁers, instead using only negations, disjunctions, and conjunctions.
13. ∃xP(x)
14. ∀xP(x)
15. ¬∃xP(x)
16. Determine whether the **argument** “If the earth is ﬂat, then you can sail off the edge of the earth. You cannot sail off the edge of the earth. Therefore, the earth is not ﬂat” is valid or not. Explain your choice.
17. Assume that the universe for *x* is all people and the universe for *y* is the set of all movies. Write the English statement using the following predicates and any needed quantifiers:

*S*(*x**y*): *x* saw *y* *L*(*x**y*): *x* liked *y* *A*(*y*): *y* won an award *C*(*y*): *y* is a comedy.

1. No comedy won an award.
2. Lois saw *Casablanca*, but didn't like it.
3. Some people have seen every comedy.
4. No one liked every movie he has seen.
5. Assume that the universe for *x* is all people and the universe for *y* is the set of all movies. Write the statement in good English, using the predicates:

*S*(*x**y*): *x* saw *y* *L*(*x**y*): *x* liked *y*.

Do not use variables in your answer

1. *y**S*(Linh*y*)
2. *y**xL*(*x**y*)
3. *x**yL*(*x**y*)

**THE END**